

Quadratic equations 2

Introduction

This leaflet will explain how quadratic equations can be solved using a formula.

1. Solving a quadratic equation using a formula.

Any quadratic equation can be solved using the **quadratic formula**.

If

$$ax^2 + bx + c = 0$$

then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

A quadratic equation has two solutions; one obtained using the positive square root in the formula, and the other obtained using the negative square root. The answers are often referred to as **roots** of the equation.

Example.

Solve the quadratic equation

$$3x^2 + 9x + 4 = 0$$

Solution

Here $a = 3$, $b = 9$ and $c = 4$. Putting these values into the quadratic formula gives

$$\begin{aligned} x &= \frac{-9 \pm \sqrt{9^2 - 4(3)(4)}}{2(3)} \\ &= \frac{-9 \pm \sqrt{81 - 48}}{6} \\ &= \frac{-9 \pm \sqrt{33}}{6} \\ &= \frac{-9 - \sqrt{33}}{6}, \frac{-9 + \sqrt{33}}{6} \\ &= -2.4574, -0.5426 \quad (4\text{dp}) \end{aligned}$$

The roots of $3x^2 + 9x + 4 = 0$ are $x = -2.4574$ and $x = -0.5426$.

Example

Solve the equation $8x^2 + 3x - 4 = 0$.

Solution

Care is needed here because the value of c is negative, that is $c = -4$.

$$\begin{aligned}x &= \frac{-3 \pm \sqrt{3^2 - 4(8)(-4)}}{(2)(8)} \\&= \frac{-3 \pm \sqrt{137}}{16} \\&= 0.5440, -0.9190 \quad (4\text{dp})\end{aligned}$$

Example

Find the roots of the quadratic equation $9x^2 + 6x + 1 = 0$.

Solution

Here $a = 9$, $b = 6$ and $c = 1$. Using the quadratic formula we have

$$\begin{aligned}x &= \frac{-6 \pm \sqrt{6^2 - 4(9)(1)}}{2(9)} \\&= \frac{-6 \pm \sqrt{36 - 36}}{18} \\&= \frac{-6 \pm \sqrt{0}}{18} \\&= -\frac{6}{18} \\&= -\frac{1}{3}\end{aligned}$$

In this example there is only one root: $x = -\frac{1}{3}$.

The quantity $b^2 - 4ac$ is called the **discriminant** of the equation. When the discriminant is 0, as in the previous Example, the equation has only one root. If the discriminant is negative we are faced with the problem of finding the square root of a negative number. Such equations require special treatment using what are called *complex numbers*.

Exercises

1. Find the roots of the following quadratic equations:

a) $x^2 + 6x - 8 = 0$, b) $2x^2 - 8x - 3 = 0$, c) $-3x^2 + x + 1 = 0$.

Answers

a) $x = -3 \pm \sqrt{17} = 1.123, -7.123$ (3dp).

b) $x = 2 \pm \frac{\sqrt{22}}{2} = 4.345, -0.345$ (3dp).

c) $x = \frac{1}{6} \pm \frac{\sqrt{13}}{6} = 0.768, -0.434$ (3dp).