

Simultaneous equations

Introduction

On occasions you will come across two or more unknown quantities, and two or more equations relating them. These are called **simultaneous equations** and when asked to solve them you must find values of the unknowns which satisfy all the given equations at the same time. On this leaflet we illustrate one way in which this can be done.

1. The solution of a pair of simultaneous equations

The solution of the pair of simultaneous equations

$$3x + 2y = 36, \quad \text{and} \quad 5x + 4y = 64$$

is $x = 8$ and $y = 6$. This is easily verified by substituting these values into the left-hand sides to obtain the values on the right. So $x = 8$, $y = 6$ **satisfy** the simultaneous equations.

2. Solving a pair of simultaneous equations

There are many ways of solving simultaneous equations. Perhaps the simplest way is **elimination**. This is a process which involves removing or eliminating one of the unknowns to leave a single equation which involves the other unknown. The method is best illustrated by example.

Example

Solve the simultaneous equations

$$\begin{array}{rcl} 3x + 2y & = & 36 \quad (1) \\ 5x + 4y & = & 64 \quad (2) \end{array} \cdot$$

Solution

Notice that if we multiply both sides of the first equation by 2 we obtain an equivalent equation

$$6x + 4y = 72 \quad (3)$$

Now, if equation (2) is subtracted from equation (3) the terms involving y will be eliminated:

$$\begin{array}{rcl} 6x + 4y & = & 72 \quad (3) \\ 5x + 4y & = & 64 \quad (2) \\ \hline x + 0y & = & 8 \end{array}$$

So, $x = 8$ is part of the solution. Taking equation (1) (or if you wish, equation (2)) we substitute this value for x , which will enable us to find y :

$$\begin{aligned} 3(8) + 2y &= 36 \\ 24 + 2y &= 36 \\ 2y &= 36 - 24 \\ 2y &= 12 \\ y &= 6 \end{aligned}$$

Hence the full solution is $x = 8, y = 6$.

You will notice that the idea behind this method is to multiply one (or both) equations by a suitable number so that either the number of y 's or the number of x 's are the same, so that subtraction eliminates that unknown. It may also be possible to eliminate an unknown by addition, as shown in the next example.

Example

Solve the simultaneous equations

$$\begin{aligned} 5x - 3y &= 26 & (1) \\ 4x + 2y &= 34 & (2) \end{aligned}$$

Solution

There are many ways that the elimination can be carried out. Suppose we choose to eliminate y . The number of y 's in both equations can be made the same by multiplying equation (1) by 2 and equation (2) by 3. This gives

$$\begin{aligned} 10x - 6y &= 52 & (3) \\ 12x + 6y &= 102 & (4) \end{aligned}$$

If these equations are now added we find

$$\begin{array}{r} 10x - 6y = 52 \quad + \quad (3) \\ 12x + 6y = 102 \quad (4) \\ \hline 22x + 0y = 154 \end{array}$$

so that $x = \frac{154}{22} = 7$. Substituting this value for x in equation (1) gives

$$\begin{aligned} 5(7) - 3y &= 26 \\ 35 - 3y &= 26 \\ -3y &= 26 - 35 \\ -3y &= -9 \\ y &= 3 \end{aligned}$$

Hence the full solution is $x = 7, y = 3$.

Exercises

Solve the following pairs of simultaneous equations:

a) $\begin{aligned} 7x + y &= 25 \\ 5x - y &= 11 \end{aligned}$, b) $\begin{aligned} 8x + 9y &= 3 \\ x + y &= 0 \end{aligned}$, c) $\begin{aligned} 2x + 13y &= 36 \\ 13x + 2y &= 69 \end{aligned}$ d) $\begin{aligned} 7x - y &= 15 \\ 3x - 2y &= 19 \end{aligned}$

Answers

a) $x = 3, y = 4$. b) $x = -3, y = 3$. c) $x = 5, y = 2$. d) $x = 1, y = -8$.