

# The laws of logarithms

## Introduction

There are a number of rules known as the **laws of logarithms**. These allow expressions involving logarithms to be rewritten in a variety of different ways. The laws apply to logarithms of any base but the same base must be used throughout a calculation.

## 1. The laws of logarithms

The three main laws are stated here:

### First Law

$$\log A + \log B = \log AB$$

This law tells us how to add two logarithms together. Adding  $\log A$  and  $\log B$  results in the logarithm of the product of  $A$  and  $B$ , that is  $\log AB$ .

For example, we can write

$$\log_{10} 5 + \log_{10} 4 = \log_{10}(5 \times 4) = \log_{10} 20$$

The same base, in this case 10, is used throughout the calculation. You should verify this by evaluating both sides separately on your calculator.

### Second Law

$$\log A - \log B = \log \frac{A}{B}$$

So, subtracting  $\log B$  from  $\log A$  results in  $\log \frac{A}{B}$ .

For example, we can write

$$\log_e 12 - \log_e 2 = \log_e \frac{12}{2} = \log_e 6$$

The same base, in this case  $e$ , is used throughout the calculation. You should verify this by evaluating both sides separately on your calculator.

### Third Law

$$\log A^n = n \log A$$

So, for example

$$\log_{10} 5^3 = 3 \log_{10} 5$$

You should verify this by evaluating both sides separately on your calculator.

Two other important results are

$$\log 1 = 0, \quad \log_m m = 1$$

The logarithm of 1 to any base is always 0, and the logarithm of a number to the same base is always 1. In particular,

$$\log_{10} 10 = 1, \quad \text{and} \quad \log_e e = 1$$

### Exercises

1. Use the first law to simplify the following.

- a)  $\log_{10} 6 + \log_{10} 3$ ,
- b)  $\log x + \log y$ ,
- c)  $\log 4x + \log x$ ,
- d)  $\log a + \log b^2 + \log c^3$ .

2. Use the second law to simplify the following.

- a)  $\log_{10} 6 - \log_{10} 3$ ,
- b)  $\log x - \log y$ ,
- c)  $\log 4x - \log x$ .

3. Use the third law to write each of the following in an alternative form.

- a)  $3 \log_{10} 5$ ,
- b)  $2 \log x$ ,
- c)  $\log(4x)^2$ ,
- d)  $5 \ln x^4$ ,
- e)  $\ln 1000$ .

4. Simplify  $3 \log x - \log x^2$ .

### Answers

- 1. a)  $\log_{10} 18$ , b)  $\log xy$ , c)  $\log 4x^2$ , d)  $\log ab^2c^3$ .
- 2. a)  $\log_{10} 2$ , b)  $\log \frac{x}{y}$ , c)  $\log 4$ .
- 3. a)  $\log_{10} 5^3$  or  $\log_{10} 125$ , b)  $\log x^2$ , c)  $2 \log(4x)$ , d)  $20 \ln x$  or  $\ln x^{20}$ , e)  $1000 = 10^3$  so  $\ln 1000 = 3 \ln 10$ .
- 4.  $\log x$ .