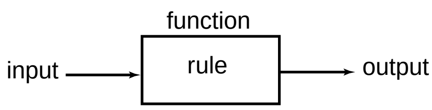
What is a function ?

Introduction

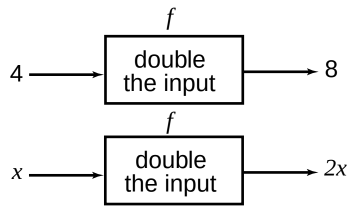
A quantity whose value can change is known as a **variable**. **Functions** are used to describe the rules which define the ways in which such a change can occur. The purpose of this leaflet is to explain functions and their notation.

1. The function rule

A function is a rule which operates on an **input** and produces an **output**. This can be illustrated using a **block diagram** such as that shown below. We can think of the function as a mathematical machine which processes the input, using a given rule, in order to produce an output. We often write the rule inside the box.



In order for a rule to be a function it must produce only a single output for any given input. The function with the rule 'double the input' is shown below.



Note that with an input of 4 the function would produce an output of 8. With a more general input, x say, the output will be $2x$. It is usual to assign a letter or other symbol to a function in order to label it. The doubling function pictured in the example above has been given the symbol f.

A function is a rule which operates on an input and produces a single output from that input.

For the doubling function it is common to use the notation

$$f(x) = 2x$$

This indicates that with an input x, the function, f, produces an output of $2x$. The input to the function is placed in the brackets after the function label 'f'. $f(x)$ is read as 'f is a function of x', or simply 'f of x', meaning that the output from the function depends upon the value of the input x.

Example
State the rule of each of the following functions:

a) $f(x) = 7x + 9$, b) $h(t) = t^3 + 2$, c) $p(x) = x^3 + 2$.

Solution
a) The rule for f is 'multiply the input by 7 and then add 9'.

b) The rule for h is 'cube the input and add 2'.

c) The rule for p is 'cube the input and add 2'.

Note from parts b) and c) that it is the rule that is important when describing a function and not the letters being used. Both $h(t)$ and $p(x)$ instruct us to 'cube the input and add 2'.

The input to a function is called its **argument**. We can obtain the output from a function if we are given its argument. For example, given the function $f(x) = 3x + 2$ we may require the value of the output when the argument is 5. We write this as $f(5)$. Here, $f(5) = 3 \times 5 + 2 = 17$.

Example
Given the function $f(x) = 4x + 3$ find a) $f(-1)$, b) $f(6)$

Solution
a) Here the argument is -1. We find $f(-1) = 4 \times (-1) + 3 = -1$.

b) $f(6) = 4(6) + 3 = 27$.

Sometimes the argument will be an algebraic expression, as in the following example.

Example
Given the function $y(x) = 5x - 3$ find

a) $y(t)$, b) $y(7t)$, c) $y(z + 2)$.

Solution
The function rule is multiply the input by 5, and subtract 3. We can apply this rule whatever the argument.

a) To find $y(t)$ multiply the argument, t, by 5 and subtract 3 to give $y(t) = 5t - 3$.

b) Now the argument is $7t$. So $y(7t) = 5(7t) - 3 = 35t - 3$.

c) In this case the argument is $z + 2$. We find $y(z + 2) = 5(z + 2) - 3 = 5z + 10 - 3 = 5z + 7$.

Exercises
1. Write down a function which can be used to describe the following rules:

a) 'cube the input and divide the result by 2', b) 'divide the input by 5 and then add 7'

2. Given the function $f(x) = 7x - 3$ find a) $f(3)$, b) $f(6)$, c) $f(-2)$.

3. If $g(t) = t^2$ write down expressions for a) $g(x)$, b) $g(3t)$, c) $g(x + 4)$.

Answers
1. a) $f(x) = \frac{x^3}{2}$, b) $f(x) = \frac{x}{5} + 7$. 2. a) 18, b) 39, c) -17

3. a) $g(x) = x^2$, b) $g(3t) = (3t)^2 = 9t^2$, c) $g(x + 4) = (x + 4)^2 = x^2 + 8x + 16$.