Maximum and Minimum Values 1

Introduction

A point on the graph of a function, $f(x)$, is a **local maximum** if, in its immediate neighbourhood, the function generates lower values to either side of the maximum than the value that it generates at the maximum point itself, as shown in Fig 1.

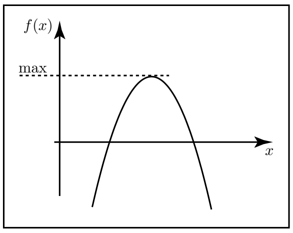

Fig 1: A Local Maximum

A point on the graph of a function, $f(x)$, is a **local minimum** if, in its immediate neighbourhood, the function generates higher values to either side of the minimum than the value that it generates at the minimum itself, as shown in Fig 2.

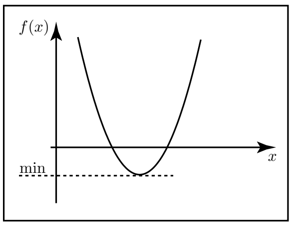

Fig 2: A Local Minimum

It is possible for a function to have more than one maximum or minimum point, as shown in Fig 3.

The points where a graph takes its maximum or minimum values may also be referred to as **turning points**.

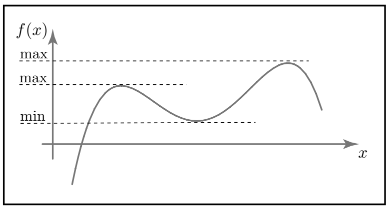

Fig 3: Two Maxima and One Minimum

Gradient

A tangent drawn at a turning point will be parallel to the horizontal x axis and the gradient at such a point will therefore be zero. This is illustrated in Fig 4.

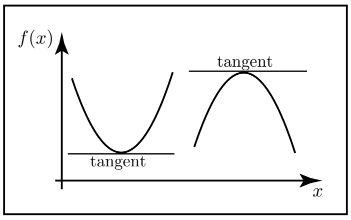

Fig 4: Tangents at a maximum and minimum

Since gradient can be determined by differentiation, it follows that at either a maximum or minimum point on the graph of $y = f(x)$, the gradient function $\frac{dy}{dx}$ will be equal to zero.

On the graph of $y = f(x)$, at a maximum or minimum point, $\frac{dy}{dx} = 0$

Example
Find the turning point of the graph of $y = x^2 - 3x + 2$

Solution
In order to locate the turning point on a graph such as $y = x^2 - 3x + 2$ it is first necessary to calculate the gradient function $\frac{dy}{dx}$.

Using the rule for differentiation of a power gives $\frac{dy}{dx} = 2x - 3$

and recalling that, at a turning point, the gradient is zero we obtain $2x - 3 = 0$

Solving $2x - 3 = 0$:
$$\begin{aligned} 2x - 3 &= 0 \\ 2x &= 3 \\ x &= 1.5 \end{aligned}$$

Calculating the corresponding value of y:
$$\begin{aligned} y &= x^2 - 3x + 2 \\ &= 1.5^2 - 3(1.5) + 2 \\ &= -0.25 \end{aligned}$$

The turning point is then located at $(1.5, -0.25)$ and consideration of the graph of $y = x^2 - 3x + 2$, Fig 5, shows that the turning point is in fact a minimum point.

We will see how the nature of the turning point can be determined in leaflet Maximum and Minimum Values 2.

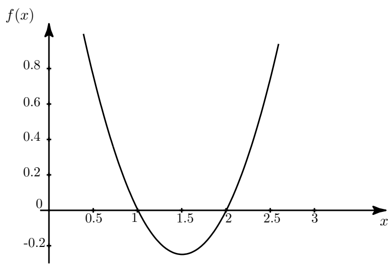

Fig 5: A graph of $y = x^2 - 3x + 2$

Exercises

For each of the following functions determine
(a) $\frac{dy}{dx}$
(b) The values of x when $\frac{dy}{dx} = 0$
(c) The corresponding values of y

1. $y = x^2 + 8x + 13$
2. $y = x^3 - 12x + 2$
3. $y = x^3 - 6x^2$
4. $y = x^4 - 2x^2$
5. $y = 0.25x + \frac{100}{x}$

Answers

1.	a) $2x + 8$	b) -4	c) -3
2.	a) $3x^2 - 12$	b) ± 2	c) $-14, 18$
3.	a) $3x^2 - 12x$	b) $0, 4$	c) $0, -32$
4.	a) $4x^3 - 4x$	b) $0, \pm 1$	c) $0, -1, -1$
5.	a) $0.25 - \frac{100}{x^2}$	b) ± 20	c) ± 10

