Maximum and Minimum Values 2

Introduction

When the location of any turning point on a graph has been determined it is useful to know whether we are dealing with a maximum or a minimum point of the function.

Determining the Nature of the Turning Points

This means deciding whether the particular turning point is a maximum or a minimum.

In the immediate neighbourhood of a maximum, the graph will have a positive gradient to the left and a negative gradient to the right, as indicated by the tangents on either side of the maximum, as shown in Fig 1.

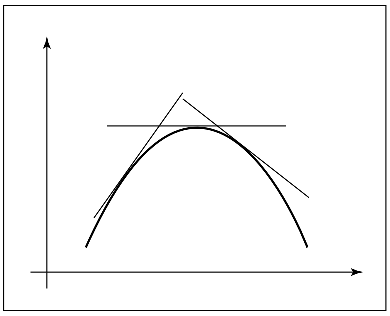
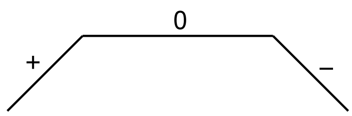


Fig 1: Gradient pattern at a maximum

The gradient pattern associated with a maximum is thus:



In the immediate neighbourhood of a minimum, the graph will have a negative gradient to the left and a positive gradient to the right, as indicated by the tangents on either side of the minimum.

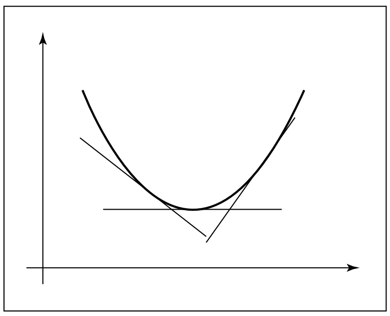
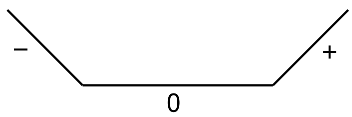


Fig 2: Gradient pattern at a minimum

The gradient pattern associated with a minimum is thus:



The nature of the turning point can therefore be determined by calculating the gradient to either side; providing we are careful to stay in a small neighbourhood close to the turning point.

Example

Determine the nature of the turning point located at $(1.5, -0.25)$ on $y = x^2 - 3x + 2$

Solution

For $y = x^2 - 3x + 2$ the gradient function is $2x - 3$ and testing the gradients at $x = 1.4$ and $x = 1.6$ leads to:

x	1.4	1.5	1.6
$\frac{dy}{dx}$	-0.2	0	0.2
	↘	—	↗

This gradient pattern indicates that the turning point is a minimum.

Exercises

For each of the following functions decide if the given turning point is a maximum or a minimum.

1. $y = x^2 + 8x + 13$ at $(-4, -3)$
2. $y = x^3 - 12x + 2$ at $(-2, 18)$
3. $y = x^3 - 6x^2$ at $(0, 0)$
4. $y = x^4 - 2x^2$ at $(1, -1)$
5. $y = 0.25x + \frac{100}{x}$ at $(20, 10)$

Answers

1. Minimum
2. Maximum
3. Maximum
4. Minimum
5. Minimum

