Rate of change

2.38 Velocity and Acceleration

Suppose that a body's position at time t is $s = f(t)$.

The body's velocity at time t is

$$v(t) = \frac{ds}{dt}.$$

The body's speed at time t is $|v(t)| = \left|\frac{ds}{dt}\right|$.

The body's acceleration at time t is

$$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}.$$

Galileo's experiments with free fall discovered that the distance a body released from rest will fall in time t is

$$s = \frac{1}{2}gt^2.$$

2.39 Example

(i) A rock breaks free from the top of the cliff.

Experiments have shown that the rock will fall

$$f(t) = 16t^2$$

feet in the first t secs.

Note $g = 32ft/sec^2$.

What is the instantaneous speed at time $t = 2$ (secs)?

$f'(t) = 32t$ and $f'(2) = 64$.

(ii) A dynamite blast blows a heavy rock straight up with a launch velocity of $160ft/sec$. It reaches a height of

$$s = 160t - 16t^2 ft$$

after t sec.

(a) How high does the rock go?

At the highest point the velocity of the rock will be 0.

$v(t) = s'(t) = 160 - 32t$.

The velocity is zero, when $160 - 32t = 0$ or $t = 5$.

$s(5) = 160 \times 5 - 16 \times 25 = 400ft$.

(b) What is the acceleration of the rock at any time t?

$a(t) = -32ft/sec^2$

(c) What are the velocity and speed of the rock when it is $256ft$ above the ground, on the way up and on the way down?

Find t such that $s(t) = 160t - 16t^2 = 256$.

Solve for $t : 16t^2 - 160t + 256 = 0$.

We get $t = 2$ and $t = 8$.

$v(2) = 160 - 32 \times 2 = 96ft/sec, \quad v(8) = -96ft/sec$

In both cases the rock's speed is $96ft/sec$.

(d) When does the rock hit the ground again?

The rock hits the ground at the positive time t for which $s = 0$.

$s(t) = 160t - 16t^2 = 16t(10 - t)$

So $s = 0$ when $t = 0$ and $t = 10$.

The rock hits the ground after $10secs$.