DUBLIN CITY UNIVERSITY

SEMESTER TWO (SAMPLE) EXAMINATION 2008 - 2009

MODULE:	MS121 IT Mathematics
COURSE:	B. Sc. in Computer Applications B. Sc. in Enterprise Computing
YEAR:	1
EXAMINERS:	Dr. R. Ryan, Dr. M. Clancy (ext. 5774) Dr. N. O'Sullivan (ext. 8219)
TIME ALLOWED:	3 hours
INSTRUCTIONS:	Answer **ALL of Question 1** and **TWO** other questions. Candidates may bring **one** A4 sheet of study notes to this examination. This A4 sheet, on which both sides may be written, must be handed in with your examination paper. Please note that where a candidate answers more than two questions from Question 2 to Question 5, then among these questions (i.e. two to five) the examiner will award marks from the two questions at which the student has obtained the highest score.
REQUIREMENTS:	Mathematical Tables

THE USE OF PROGRAMMABLE OR TEXT STORING CALCULATORS IS EXPRESSLY FORBIDDEN

DO NOT TURN OVER THIS PAGE UNTIL INSTRUCTED TO DO SO.

QUESTION 1 **All parts carry 5 marks**

(i) Express the repeating decimal $3.1\underbrace{422}\underbrace{422}\underbrace{422}\ldots$ as a fraction.

(ii) Let X and Y be subsets of some universal set U. Use De Morgan's Laws to show that $\overline{(X \cap \overline{Y})} \cap Y$ is equal to one of the following:

(a) $\overline{X} \cap Y$ **(b)** $\overline{X} \cup Y$ **(c)** X **(d)** Y **(e)** $\emptyset$

(iii) Let $\boldsymbol{\mathcal{L}}$ be the set of all **lines** in the xy-plane and let $\Re$ be the relation on $\boldsymbol{\mathcal{L}}$ given by

$$\boxed{\ell_1 \,\Re\, \ell_2} \iff \boxed{\text{the line } \ell_1 \text{ intersects the line } \ell_2}.$$

Is it true that the relation $\Re$ is an equivalence relation? Justify your answer.

(iv) In the case of a real variable x, let $f(x) = \dfrac{1}{x^2 - 1}$ and let $g(x) = x^4$. Now calculate $(g \circ f)(x)$ and determine the largest subset $X \subseteq \mathbb{R}$ for which $(g \circ f)(x)$ is defined.

(v) Sketch the graph of the polynomial $p(x) = (x+3)^2(x+1)x^3(x-2)$.

(vi) If two cards are drawn, without replacement, from a standard fair deck of 52 cards, calculate the probability that the pair consists of a **black card** and a **diamond** (in any order).

(vii) Evaluate $\displaystyle \lim_{x \to -2} \left(\frac{x^3 + 8}{x^2 - x - 6} \right)$.

(viii) Find the derivative of $f(x) = \dfrac{x^3 - 2x}{x + 5}$.

(ix) Find the derivative of $f(x) = \left[\ln(x^2 + 1) - \dfrac{1}{x} \right]^{10}$.

(x) Determine the integral $\displaystyle \int_0^1 x(x^2 - 1)^{10}\, dx$.

(xi) Determine the integral $\displaystyle \int e^{2x} e^{3x}\, dx$.

(xii) Suppose that X is a normal random variable with $X \sim N(200, 25)$. Calculate

$$P(190 \leq X \leq 205).$$

QUESTION 2

(a) (i) Explain what is meant by a **left inverse** of a function $f : X \to Y$.

[2 marks]

(ii) Explain what it means for a function $f : X \to Y$ to be **injective**.

[2 marks]

(iii) If $f : X \to Y$ has a left inverse, prove that f is injective.

[4 marks]

(iv) In $\mathbb{Z}_{14}$ (that is, in the the set of equivalence classes of integer remainders on division by 14) let $[k]$ denote the equivalence class of $k \in \mathbb{Z}$. For any $[n]$ and $[m] \in \mathbb{Z}_{14}$, explain how the product $[n].[m]$ is defined. Is it true that the map

$$f_3 : \mathbb{Z}_{14} \to \mathbb{Z}_{14} : [n] \mapsto [3].[n]$$

is injective? Does f_3 have a left inverse? Justify your answer in both cases.

[4 marks]

(b) Prove by induction on $n \in \mathbb{N}$ that

$$\sum_{j=1}^{n} \left(\frac{1}{j^2} \right) \leq 2 - \frac{1}{n}.$$

[8 marks]

QUESTION 3

(a) In the following P and Q will be proposition.

(i) State the **converse** of the proposition $(P \Rightarrow Q)$.

[2 marks]

(ii) Is it true, in general, that the truth of the proposition $(P \Rightarrow Q)$ implies the truth of its converse? Justify your answer.

[3 marks]

(b) If x and y are real variables, is the following proposition true:

$$(\forall y)(\exists x)[x^2 = y^2 + 1] \, ?$$

Justify your answer.

[3 marks]

(c) (i) Explain what is meant by the statement:

The sets $E_1, E_2, \ldots, E_n$ form a **partition** of the set S

.

[2 marks]

(ii) At the end of an examination paper students were asked to tick one of two boxes indicating whether or not they believed they would score above the average mark for the class on that examination. After the examination papers were graded it was found that while 70% of students did indeed score above the average only 60% of the class indicated the belief that they would do so. However, of those who did in fact score above the class average it transpired that 80% indicated the belief that they would do so. If a student is picked at random from this class find **the probability that they indicated the belief that they would score above average if in fact they did not do so**.

Hint: Consider the events;

$A = \{\text{the student actually scored above average}\}$ and

$$I = \left\{ \begin{array}{c} \text{the student indicated the belief} \\ \text{that they would score above average} \end{array} \right\}.$$

If $\bar{A}$ denotes the complement of A, then $I = (A \cap I) \cup (\bar{A} \cap I)$

[10 marks]

QUESTION 4

In the case of the function

$$f : \boldsymbol{R} \setminus \{0\} \to \boldsymbol{R} : x \mapsto \frac{(x+1)(x-3)}{x^2},$$

do the following:

(a) Determine the intervals (if there are any) on which f is positive and the intervals on which f is negative.

[3 marks]

(b) Determine the behaviour of $f(x)$ as $x \to +\infty$ and as $x \to -\infty$.

[3 marks]

(c) At each $x_0 \in \boldsymbol{R}$ where $f(x_0)$ is **not** defined determine $\lim_{x \uparrow x_0} f(x)$ and $\lim_{x \downarrow x_0} f(x)$.

[4 marks]

(d) Determine the intervals (if there are any) on which f is increasing and the intervals on which f is decreasing.

[6 marks]

(e) Sketch the graph of f, that is, sketch the curve $y = f(x)$.

[4 marks]

QUESTION 5

In a quality assurance study, 10 bottles of lemonade, each nominally containing 1 litre, are selected and the true volume of lemonade contained in each is measured. The results, in mls, are

$$997, \quad 1003, \quad 999, \quad 988, \quad 1006,$$
$$1005, \quad 1007, \quad 1009, \quad 994, \quad 1002.$$

(a) Calculate the sample mean and sample standard deviation for these measurements.

[6 marks]

(b) Construct the 99% confidence interval for the true mean μ using a distribution appropriate for this small sample.

[7 marks]

(c) A further study is undertaken, in which 150 sample bottles are used. The sample mean and standard deviation are found to be $\bar{x} = 998$ and $s = 4$ respectively. Using an appropriate distribution, calculate the 99% confidence interval for the true mean.

[7 marks]