Proposition: The square root of 2 is **not** a rational number.

Proof (by contradiction): Recall that a proof by contradiction works by assuming the contrary statement to be true and then by showing that this assumption leads to a contradiction.

In this case, the contrary statement is that $\sqrt{2}$ is rational. Thus, we assume that we can write

$$\sqrt{2} = \frac{m}{n} \tag{1}$$

where both m and n are natural numbers. In addition, after cancellation of all common factors, we may further assume that

$$m \text{ and } n \text{ have no common factors.} \tag{2}$$

It follows from (1) above that

$$2 = \frac{m^2}{n^2} \tag{3}$$

so that

$$m^2 = 2\,n^2. \tag{4}$$

Equation (4) means that m^2 is an even number and, therefore, m itself must also be even. That is, we can write

$$m = 2\,k, \quad \text{for some natural number } k. \tag{5}$$

Putting $m = 2\,k$ in equation (3) we find that

$$2 = \frac{(2\,k)^2}{n^2} \tag{6}$$

or that

$$2 = \frac{4\,k^2}{n^2}. \tag{7}$$

so that,

$$n^2 = 2\,k^2. \tag{8}$$

That is, n^2 is an even number and, therefore, n itself must also be even.

Putting this together with the above, i.e. with equation (5), we have shown that

both m and n are even, (that is have 2 as a common factor).

But this latter statement is in direct contradiction of statement (2) above!

QED