

MS 121 IT Mathematics Solutions Test 3

1. The limit as $x \rightarrow -2$ of the function

$$f(x) = \frac{x^3 + 5x^2 - 12}{x^3 + 2x^2 + 3x + 6}$$

is equal to:

- (A) ∞ (B) $\frac{4}{7}$ (C) 8 (D) $-\frac{8}{7}$ (E) None of the previous.

2. The limit as $x \rightarrow -\infty$ of the function

$$f(x) = \frac{29x^3 - 7x^2 + 2x - 11}{5x^4 + 6x^2 - 5x + 3}$$

is equal to:

- (A) 0 (B) $-\infty$ (C) ∞ (D) $\frac{29}{5}$ (E) None of the previous.

3. The derivative of the function $f(x) = 10x^6 - \frac{6}{x^6}$ is:

- (A) $60x^5 + \frac{36}{x^5}$ (B) $60x^5 - \frac{36}{x^7}$ (C) $60x^5 + \frac{36}{x^7}$
(D) $60x^5 - \frac{1}{x^7}$ (E) None of the previous.

4. The derivative of the function $f(x) = (6x^3 + 2x)^{-3}$ is:

- (A) $-3(18x^2 + 2)(6x^3 + 2x)^{-4}$ (B) $-3(18x^2 + 2)(6x^3 + 2x)^{-2}$
(C) $-3(6x^3 + 2x)^{-4}$ (D) $-3(6x^3 + 2x)^{-2}$ (E) None of the previous.

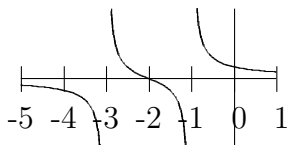
5. Evaluate the following integral: $\int_{-1}^1 x^4 - \frac{1}{x^4} dx$.

- (A) 0 (B) $\frac{16}{15}$ (C) $-\frac{4}{15}$ (D) $\frac{4}{5}$ (E) None of the previous.

6. The function $f(x) = \frac{x^2 - 1}{x^2 + 3}$ is increasing on the interval(s):

- (A) $(-\infty, \infty)$ (B) $(0, \infty)$ (C) $(-\infty, -1)$ and $(1, \infty)$
 (D) $(-\infty, 0)$ (E) None of the previous.

7. The following curve in the xy -plane



most closely represents the graph of $f(x) =$:

- (A) $\frac{x^2}{(x+3)(x+1)}$ (B) $\frac{x}{(x+3)(x+1)}$ (C) $\frac{x+2}{(x+3)(x+1)}$
 (D) $\frac{x+2}{(x-3)(x-1)}$ (E) None of the previous.

8. On the interval $[-1, 2]$ the function $f(x) = \frac{x+5}{x-6}$ takes its maximum value at:

- (A) $x = -1$ (B) $x = 0$ (C) $x = 1$ (D) $x = 2$ (E) None of the previous.

9. Determine the following integral:

$$\int \frac{x^2 - 2}{(x^3 - 6x + 8)^3} dx.$$

- (A) $\frac{-1}{6(x^3 - 6x + 8)^2} + C$ (B) $\frac{-1}{2(x^3 - 6x + 8)^2} + C$ (C) $\frac{-1}{12(x^3 - 6x + 8)^4} + C$
 (D) $\frac{-1}{4(x^3 - 6x + 8)^4} + C$ (E) None of the previous.

10. Evaluate the following integral:

$$\int_2^4 x(x-3)^6 dx.$$

- (A) $\frac{2}{5}$ (B) 0 (C) $\frac{2}{7}$ (D) $\frac{4}{7}$ (E) None of the previous.

1. Both the numerator and denominator evaluated at $x = -2$ are zero, so $(x + 2)$ is a factor of both the numerator and denominator:

$$\lim_{x \rightarrow -2} \frac{x^3 + 5x^2 - 12}{x^3 + 2x^2 + 3x + 6} = \lim_{x \rightarrow -2} \frac{(x + 2)(x^2 + 3x - 6)}{(x + 2)(x^2 + 3)} = \lim_{x \rightarrow -2} \frac{x^2 + 3x - 6}{x^2 + 3} = \frac{4 - 6 - 6}{4 + 3} = -\frac{8}{7} \quad (D)$$

2. Divide by the highest power of x in the denominator, which is x^4 :

$$\lim_{x \rightarrow -\infty} \frac{29x^3 - 7x^2 + 2x - 11}{5x^4 + 6x^2 - 5x + 3} = \lim_{x \rightarrow -\infty} \frac{29/x - 7/x^2 + 2/x^3 - 11/x^4}{5 + 6/x^2 - 5/x^3 + 3/x^4} = 0 \quad (A).$$

3. Use the following

$$\frac{d}{dx} (x^n) = nx^{n-1},$$

for $n \neq 0$.

$$\frac{d}{dx} \left(10x^6 - \frac{6}{x^6} \right) = 60x^5 + \frac{36}{x^7} \quad (C).$$

4. Use the Power chain rule

$$\frac{d}{dx} (u^n) = nu^{n-1} \frac{du}{dx}.$$

Here $u = 6x^3 + 2x$ and $n = -3$ so

$$f'(x) = -3(18x^2 + 2)(6x^3 + 2x)^{-4} \quad (A).$$

5. Use the following

$$\int_a^b x^n dx = \left[\frac{x^{n+1}}{n+1} \right]_a^b$$

for $n \neq -1$.

$$\int_{-1}^1 x^4 - \frac{1}{x^4} dx = \left[\frac{x^5}{5} + \frac{1}{3x^3} \right]_{-1}^1 = \frac{1}{5} + \frac{1}{3} - \left(-\frac{1}{5} - \frac{1}{3} \right) = \frac{16}{15} \quad (B).$$

6. The function $f(x)$ is increasing whenever $f'(x) > 0$.

$$f'(x) = \frac{2x(x^2 + 3) - 2x(x^2 - 1)}{(x^2 + 3)^2} = \frac{8x}{(x^2 + 3)^2} > 0$$

if $x > 0$. So the function is increasing on $(0, \infty)$ (B).

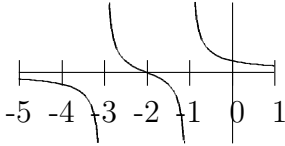
7. The graph crosses the x -axis at $x = -2$ so $f(-2) = 0$, the only answer which satisfies this is (C) (and possibly (E)). We can see from the graph that f is undefined when $x = -3$, and $x = -1$. Also

$$\lim_{x \rightarrow \infty} \frac{x + 2}{(x + 3)(x + 1)} = 0 = \lim_{x \rightarrow -\infty} \frac{x + 2}{(x + 3)(x + 1)}$$

$$\lim_{x \uparrow -3} \frac{x + 2}{(x + 3)(x + 1)} = -\infty = \lim_{x \uparrow -1} \frac{x + 2}{(x + 3)(x + 1)}$$

$$\lim_{x \downarrow -3} \frac{x + 2}{(x + 3)(x + 1)} = \infty = \lim_{x \downarrow -1} \frac{x + 2}{(x + 3)(x + 1)}$$

as in the diagram so that C is the answer.



8. The function f will take its maximum values either at the endpoints, at x_0 where $f'(x_0) = 0$, or at x_0 where f' is undefined.

$$f'(x) = \frac{(x-6) - (x+5)}{(x-6)^2} = -\frac{11}{(x-6)^2}.$$

So $f'(x)$ is defined and $f'(x) < 0$, for all $x \in [-1, 2]$. As $f'(x) < 0$ we can see that f is decreasing and hence takes its maximum value at the lefthand endpoint $x = -1$ (A). Alternatively evaluate f at the endpoints of the interval

$$f(-1) = -4/7 > f(2) = -3/4.$$

9. Use the following:

$$\int u' u^n dx = \int u^n du = \frac{u^{n+1}}{n+1} + C.$$

Here $u = x^3 - 6x + 8$, $u' = 3x^2 - 6 = 3(x^2 - 2)$ and $n = -3$. So

$$\int \frac{x^2 - 2}{(x^3 - 6x + 8)^3} dx = \frac{1}{3} \int \frac{3x^2 - 6}{(x^3 - 6x + 8)^3} dx = \frac{-1}{6(x^3 - 6x + 8)^2} + C \quad (A).$$

10. Use integration by parts:

$$\int_a^b uv' dx = [uv]_a^b - \int_a^b u'v dx.$$

Here $u = x$, $v' = (x-3)^6$ so $u' = 1$ and $v = \frac{(x-3)^7}{7}$.

$$\begin{aligned} \int_2^4 x(x-3)^6 dx &= \left[\frac{x(x-3)^7}{7} \right]_2^4 - \int_2^4 \frac{(x-3)^7}{7} dx = \left[\frac{x(x-3)^7}{7} - \frac{(x-3)^8}{56} \right]_2^4 \\ &= \frac{4}{7} - \frac{1}{56} - \left(-\frac{2}{7} - \frac{1}{56} \right) = \frac{6}{7} \quad (E). \end{aligned}$$